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Detection of Isomorphism among planar kinematic chains

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Abstract— The structural synthesis of kinematic chains involves complete creation of the set of kinematic chains, which is followed by the test for isomorphism to discard duplicate chains. Hamming number technique a unique process is adopted for the test of isomorphism. The concept is derived from the digital communication theory in which it dependency/connectivity is defined by on 0 or 1 through the connectivity matrix. The proposed method also has the capability of identifying the possible mechanisms from a particular chain and also its applicability is upto 10-link kinematic chains. The method is computationally simple and efficient compared to the other polynomial techniques

Index Terms— Kinematic chain, Link, Isomorphism, connectivity, polynomial, mechanism, digital communication.

1 Introduction

In a mechanism design problem, systematic steps are type synthesis, structural/number synthesis and dimensional synthesis. Structural analysis and synthesis of the Kinematic Chain (KC) and mechanism has been the subject of a number of studies in recent years. One important aspect of structural synthesis is to develop the all-possible arrangements of KC and their derived mechanisms for a given number of links, joints and degree of freedom, so that the designer has the liberty to select the best or optimum mechanisms according to his requirements. In the course of development of KC and mechanisms, duplication may be possible. One very important problem encountered during structural synthesis of chains is the detection of possible isomorphism among planar chains with simple as well as multiple joints.

A Linkage Characteristics Polynomial was defined by Yan, H.S. and Hall, A.S. [5], which is the characteristics polynomial of the adjacency matrix of the kinematic graph of the kinematic chain. A rule which all the coefficients of the characteristic polynomial of a kinematic chain can be identified by inspection, based on the interpretation of a graph determinant, was derived and presented.

This inspection rule interprets the topological meaning behind each characteristics coefficient, and might have some interesting possible uses in studies of the structural analysis and synthesis of kinematic chains.

Several assembly theorems were presented and derived by Yan, H.S. and Hall, A.S. [6] for obtaining the Linkage Characteristic Polynomial for a complex chain through a series of steps involving the known polynomials for subunits of the chain, are derived and presented. These theorems give insight into how the topological information concerning the linkage is stored in the polynomial and might contribute to the automated recognition of linkage structure in generalized computerized design programs. Based on graph theory, the characteristics polynomial cannot characterize the graph up to Isomorphism. However, for practical applications in the field of linkage mechanisms, it is extremely likely that the characteristic polynomials are unique for closed connected kinematic chains without any over constrained sub chains.

Mruthyunjaya, T.S. [8] made an effort to develop a fully computerized approach for structural synthesis of kinematic chains. The steps involved in the method of structural synthesis based on transformation of binary chains, have been recast in a format suitable for implementation on a digital computer. The methodology thus evolved has been combined with the algebraic procedures for structural synthesis and analysis of simple jointed kinematic chains with a degree of freedom ≥0.

The test based on comparison of the Characteristic Coefficients of the adjacency matrices of the corresponding graphs for detection of Isomorphism in kinematic chains has been shown to fail in the case of two pairs of ten links, simple jointed chains, one pair corresponding to single freedom chains and the other pair corresponding to three freedom chains. An assessment of the merits and demerits of available methods for detection of Isomorphism in graphs and kinematic chains was presented by Mruthyunjaya, T.S. and Balasubramanian, H.R. [9], keeping in

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view the suitability of the methods for use in computerized structural synthesis of kinematic chains. A new test based on the Characteristic Coefficients of the "degree" matrix of the corresponding graph is proposed for detection of isomorphism in kinematic chains. The new test was found to be successful in the case of a number of examples of graphs where the test based on Characteristic Coefficients of adjacency matrix fails. It has also been found to be successful in distinguishing the structures of all known simple-jointed kinematic chains in the categories of (a) single-freedom chains with upto 10 links, (b) two-freedom chains with upto 9 links and (c) three-freedom chains with upto 10 links.

Ambedkar, A.G. and Agrawal, V.P. [10] explained the concept of minimum code and discussed its properties relevant to kinematic chains. An algorithm, based on one method available in the graph theoretic literature in chemistry, is elaborated to demonstrate its applicability in establishing minimum code of kinematic chains with simple joints. Minimum code, being unique for kinematic chains, is suitable for testing isomorphism. The decidability of this code positively indicates its possiblity in cataloguing (storage and retrieval) of kinematic chains and mechanisms.

Min code, as canonical number, was shown to give a unique number for kinematic chains with simple joints. Ambedkar, A.G. and Agrawal, V.P. [11] suggested a method to identify mechanisms, path generators and function generators through a set of identification numbers. The concept of min code is also shown to be effective in revealing the topology of kinematic chains and mechanisms consisting of (a) different types of lower pairs, and/or (b) simple and multiple joints.

Loop connectivity properties of multi-loop kinematic chains are used to develop a hierarchical classification scheme of kinematic structures. A loop-loop permanent matrix is defined by Agrawal, V.P. and Rao, J.S. [12] leading to mathematical equation (permanent function) and an identification set which is an invariant of a grouping of chains. The scheme reduces computer time and effort in the optimum selection of a kinematic structure from a large family of kinematic chains using a computer-aided design program.

Many methods were available to the kinematicians to detect isomorphism among chains and among inversions but each has its own shortcomings. Rao, A.C. [14] presented a novel approach of Hamming Number Technique which is both reliable and simple. Use is made of the Hamming number, a concept borrowed from digital communication theory. The connectivity matrix of various links, a matrix of zeroes and ones, is first formed and Hamming number matrix is computed. The link Hamming string-which is defined as the string obtained by concatenating the link Hamming number and the frequency of individual Hamming numbers in that row-is then formed. Finally, the chain Hamming string, defined as the string obtained by the concatenation of the chain Hamming number and the link Hamming string, is an excellent test for the isomorphism among chains. Also, the link Hamming String of every link together with those of its neighbours is an excellent test for isomorphism among the inversions of a given chain.

Topology of kinematic chains is useful in comparing them for the structural- error point of view and an attempt was made by Rao, A.C. and Rao, C.N. [15] in this direction. The method reported, however, fails to compare the chains which consist of the same number and type of links and joints; ternary-binary, ternary-quaternary, etc. but differ in loop formation only. Further, comparison of the loop hamming values of links and chains is expected to be the simplest and positive test for isomorphism.

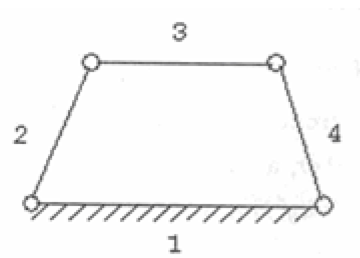
2 HAMMING NUMBER

The *Hamming Number* for any two codes each with *n* digits has been defined as the total number of bits in which the two codes differ. Applying this definition to the rows *i* and *j* of C, it becomes:

$$hij = \sum_{k=1}^{n} Sk,$$

where Sk $= 0,$ if Cik = Cjk
$= Cik + Cjk,$ otherwise

The *Hamming Number* between any two rows of size *n* can be any positive integer from *n* (if all the digits are different), down to 0 (if the two rows are exactly identical).
To put this in plain English, *Hamming Number* of any two rows is the sum of all the scores for each of the columns of those rows. A score, in turn, is defined
as (i) the sums of the individual elements if they are unequal and (ii) zero if the elements are equal. Thus if elements are (0 and 0) or (1 and 1), the score is 0. But if they are (1 and 0) or (0 and 1), the score is (0+1=1+0=1). In Boolean algebra terminology this score goes by the name XOR for exclusive OR.
Applying this definition to the chain of Fig.2.1,



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it is- $h12 = h14 = 1+1+1+1 = 4$ whereas $h13 = 0+0+0+0 = 0$

In the same manner *Hamming Number* for all other pairs of rows are calculated and one obtains the *Hamming Number Matrix* as *H = [hij]*. Thus for the four-bar chain of Fig. 2.1, the *Hamming Matrix* is: **H =**

0	4	0	4
4	0	4	0
0	4	4	4
4	0	4	0

The *Hamming* Matrix is also a square, symmetric matrix and has zeroes all along its leading diagonal. However unlike the Connectivity *Matrix it* contains digits which could be larger than unity.

2.1 Definitions and Terms

- **Link Hamming Number (L.H.N.)** for any link *i* is the sum of all the elements in the i^{th} row of the Hamming Matrix.Thus the link *Hamming Number* for link 1 of Fig. 5.1 is 8 (=0+4+0+4), so also for all the other links.
- **Chain Hamming Number (C.H.N.)** for any chain is the sum of the entire link *Hamming Numbers* of that chain. It also works out to be the sum of all the elements of the *Hamming Matrix* for that chain.The *Chain Hamming Number* for the four-bar chain is 32 (=8+8+8+8).
- **Link Hamming String** for any link *i* is the string obtained by concatenating (a) the link *Hamming Number* of *i* with (b) the frequency of occurrence, of all the integers from *n* down to *0*, in the *Hamming Numbers* of that row *i*. For the example considered, the *link Hamming String* for link 1 is 8, 20002, implying that link *Hamming Number* is 8 and comprises of two 4s, no 1s and two 0s.
  - The *Link Hamming Strings* for the four links are:
    - 1: 8, 20002
    - 2: 8, 20002
    - 3: 8, 20002
    - 4: 8, 20002
  - It may be observed that all the four links 1,2,3,4 have the same *Link Hamming Number.*
- **Chain Hamming String** is defined as the concatenation of the (i) Chain Hamming Number and (ii) Link Hamming String placed in decreasing order of magnitude. For the example being considered here, the Chain Hamming Number is 32 and the *Chain Hamming String* is:

32; 8, 20002; 8, 20002; 8, 20002; 8, 20002

3 ISOMORPHISM FOR N-LINK PLANAR KINEMATIC CHAINS

3.1 Isomorphism among kinematic chains

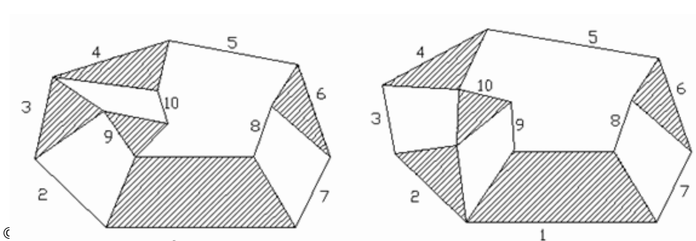
Two chains are isomorphic, if their links & adjacent relationship between the links are one to one correspondent

Two kinematic chains are considered to be isomorphic, if a simple relabeling of the links can show them to be identical. This is because the structure of the kinematic chain is independent of labeling operation

3.2 Detection of Isomorphism among kinematic chains

The Chain Hamming String is a definitive test for isomorphism among the kinematic chains. This implies that if two chains are known to be isomorphic, their Chain Hamming String should be identical and vice-versa. Secondly, if two chains are non-isomorphic their Chain Hamming String should differ at some position or other.

The example concerns two kinematic chains with 10 bars and single-degree of freedom as shown in Fig.3.2 (a) and Fig. 3.2 (b). The task is to examine whether these two chains are isomorphic.



(a) (b)

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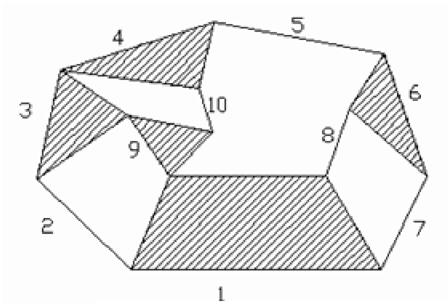
Figure: 3.2(a) ,(b) 10-Link kinematic chains



(a)

Figure: 3.2(a) 10-link kinematic chain-1

The detection of isomorphism involves the following steps

STEP 1 --- CONNECTIVITY MATRIX(C)

Link	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	0	0	0	0	1	1	1	0
2	1	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	1	0
4	0	0	1	0	1	0	0	0	0	1
5	0	0	0	1	0	1	0	0	0	0
6	0	0	0	0	1	0	1	1	0	0
7	1	0	0	0	0	1	0	0	0	0
8	1	0	0	0	0	1	0	0	0	0
9	1	0	1	0	0	0	0	0	0	1
10	0	0	0	1	0	0	0	0	1	0

STEP 2 --- HAMMING MATRIX (H)

Link	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**L.H.N.**
1.	0	6	3	7	6	3	6	6	7	4	48
2.	6	0	5	3	4	5	2	2	1	4	32
3.	3	5	0	6	3	6	5	5	6	1	40
4.	7	3	6	0	5	4	5	5	2	5	42
5.	6	4	3	5	0	5	2	2	5	2	34
6.	3	5	6	4	5	0	5	5	6	5	44
7.	6	2	5	5	2	5	0	0	3	4	32
8.	6	2	5	5	2	5	0	0	3	4	32
9.	7	1	6	2	5	6	3	3	0	5	38
10.	4	4	1	5	2	5	4	4	5	0	34

∑ L.H.N. = C.H.N. = 376

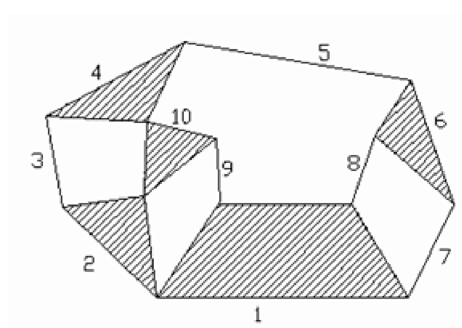
STEP 3 --- LINK HAMMING STRING

Link	**Link Hamming String**
1	48, 10021042
2	32, 11212210
3	40, 11020330
4	42, 10111411
5	34, 10311310
6	44, 10011520
7	32, 20211310
8	32, 20211310
9	38, 11120221
10	34, 11104300

STEP 4 --- CHAIN HAMMING STRING (C.H.S.)

C.H.S. **=** 376; 48, 10021042; 44, 10011520; 42, 10111411; 40, 11020330; 38, 11120221; 34, 11104300; 34, 10311310; 32, 20211310; 32, 20211310; 32, 11212210

For chain -2 the following steps are caluculated for detection of isomporhism with the chain-1



(b)

Figure: 3.2(b) 10-link kinematic chain-2

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STEP 1 --- CONNECTIVITY MATRIX (C)

Link	1	2	3	4	5	6	7	8	9	10
1.	0	1	0	0	0	0	1	1	1	0
2.	1	0	1	0	0	0	0	0	0	1
3.	0	1	0	1	0	0	0	0	0	0
4.	0	0	1	0	1	0	0	0	0	1
5.	0	0	0	1	0	1	0	0	0	0
6.	0	0	0	0	1	0	1	1	0	0
7.	1	0	0	0	0	1	0	0	0	0
8.	1	0	0	0	0	1	0	0	0	0
9.	1	0	0	0	0	0	0	0	0	1
10.	0	1	0	1	0	0	0	0	1	0

STEP 2 --- HAMMING MATRIX (H)

Link	1	2	3	4	5	6	7	8	9	10	L.H.N.
1.	0	7	4	7	6	3	6	6	6	3	48
2.	7	0	5	2	5	6	3	3	1	6	38
3.	4	5	0	5	2	5	4	4	4	1	34
4.	7	2	5	0	5	4	5	5	3	6	42
5.	6	5	2	5	0	5	2	2	4	3	34
6.	3	6	5	4	5	0	5	5	5	6	44
7.	6	3	4	5	2	5	0	0	2	5	32
8.	6	3	4	5	2	5	0	0	2	5	32
9.	6	1	4	3	4	5	2	2	0	5	32
10	3	6	1	6	3	6	5	5	5	0	40

C.H.N. = ∑ L.H.N. = 376

STEP 3 --- LINK HAMMING STRING

Link	Link Hamming String
1	48, 10021042
2	32, 11120221
3	34, 11104300
4	42, 10111411
5	34, 10311310
6	44, 10011520
7	32, 20211310
8	32, 20211310
9	32, 11212210
10	40, 11020330

STEP 4 --- CHAIN HAMMING STRING (C.H.S.)

C.H.S. = 376; 48, 10021042; 44, 10011520;
42, 10111411; 40, 11020330;
38, 11120221; 34, 11104300;
34, 10311310; 32, 20211310;
32, 20211310; 32, 11212210

Now even a cursory glace reveals that Fig. 3.2(a) and (b) have same C.H.S., hence they are isomorphic.

Therefore the chains-1 & 2 are said to be isomorphic chains

5 CONCLUSION

The Link Hamming String of every link together with those of its neighbours is an excellent test for isomorphism among the inversions of given chains. These twin claims have been verified on a computer for all six, eight and ten-bar chains with one degree of freedom as well as ten-bar chains with three-degrees of freedom. It is felt that the greatest advantage of this method is that the Chain Hamming String reveals at a glance, without much additional computation, how many inversions are possible out of a given chain. But computations become very long in case of large KC as Link Hamming String and Chain Hamming String are calculated using the Connectivity Matrix, Hamming Matrix, Link Hamming Number and Chain Hamming Number. So it takes a lot of effort & time and is difficult to compute all these things.

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